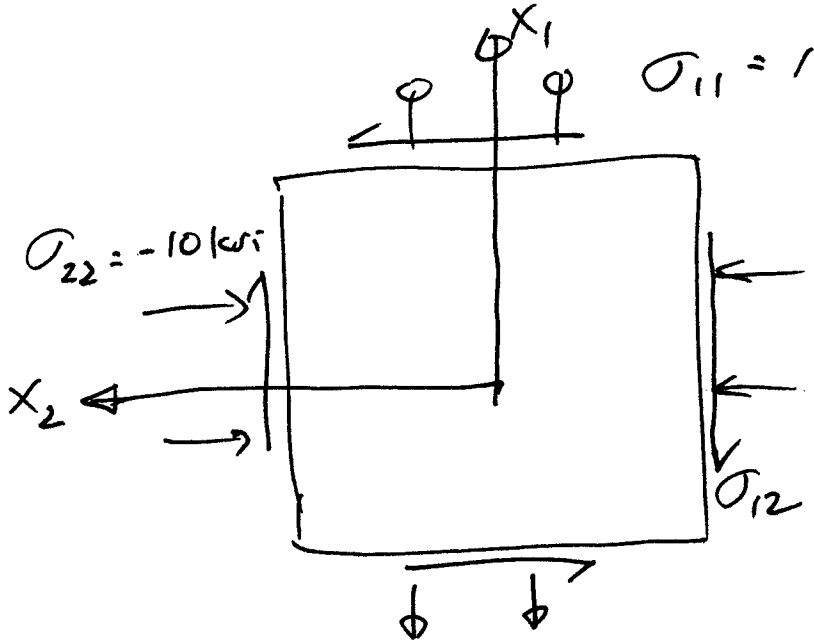
PAL
10/31/07

Unified Engineering Problem Set
Week 10 Fall, 2007
SOLUTIONS

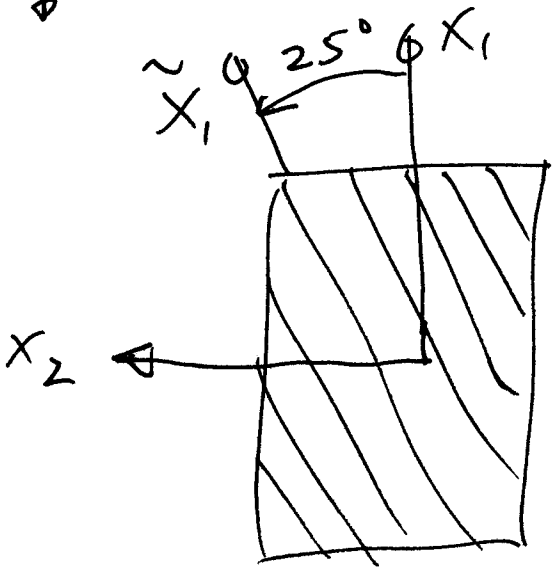
M10.1



$\sigma_{11} = 15$ ksi

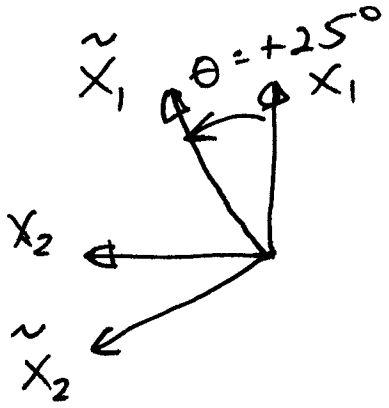
$\sigma_{22} = -10$ ksi

$\sigma_{12} = -5$ ksi



→ To find the stress state relative to the "fiber axes", align the transformed axis system with its $\tilde{X}_1$ axis along the fiber direction.

So for the $\theta = +25^\circ$ case:



→ Then use the transformation equations (in 2-D):

$$\tilde{\sigma}_{11} = \cos^2\theta\,\sigma_{11} + \sin^2\theta\,\sigma_{22} + 2\cos\theta\sin\theta\,\sigma_{12}$$

$$\tilde{\sigma}_{22} = \sin^2\theta\,\sigma_{11} + \cos^2\theta\,\sigma_{22} - 2\cos\theta\sin\theta\,\sigma_{12}$$

$$\tilde{\sigma}_{12} = -\sin\theta\cos\theta\,\sigma_{11} + \cos\theta\sin\theta\,\sigma_{22} + (\cos^2\theta - \sin^2\theta)\,\sigma_{12}$$

for these axes and $\theta = 25^\circ$:

$\sin\theta = 0.422 \Rightarrow \sin^2\theta = 0.179$

$\cos\theta = 0.906 \Rightarrow \cos^2\theta = 0.821$

and: $\sin\theta\cos\theta = 0.382$

Plugging into the transformation equations gives:

$$\tilde{\sigma}_{11} = (0.821)(15\text{ksi}) + (0.179)(-10\text{ksi}) + (0.765)(-5\text{ksi})$$

$$\tilde{\sigma}_{22} = (0.179)(15\text{ksi}) + (0.821)(-10\text{ksi}) - (0.765)(-5\text{ksi})$$

$$\tilde{\sigma}_{12} = (-0.382)(15\text{ksi}) + (0.382)(-10\text{ksi}) + (0.821-0.179)(-5\text{ksi})$$

working through this gives for $\theta = +25°$:

$$\sigma_{11} = 6.7\text{ ksi}$$
$$\sigma_{22} = -1.7\text{ ksi}$$
$$\sigma_{12} = -12.8\text{ ksi}$$